

Nematicons in liquid crystals with negative dielectric anisotropy

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We report a theoretical and experimental work on the nematicon in the planar cell containing the nematic liquid crystal with negative dielectric anisotropy, aligned homeotropically in the presence of an externally applied voltage. The formation of the soliton is resulted from the balance between the linear diffraction and the nonlocal nonlinearity due to molecular reorientation. © 2014 Optical Society of America

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Nematicons, spatial optical solitons in nematic liquid crystals (NLC), have been the subject of intense theoretical and experimental studies over the past two decades [1]. The pioneering work on nematicons was reported in 1993 by Braun *et al.* [2]. They investigated the strong self-focusing of a laser beam in NLC in various geometries, from which they recognized the importance of molecular reorientation and anchoring at the boundaries. Subsequently, in 1998, Warenghem *et al.* observed the beam self-trapping in capillaries filled with dye-doped NLC [3]. In the same year, Karpierz *et al.* observed the same phenomenon in planar cells with homeotropically aligned NLC [4]. They lowered the required power of observing nematicons to milliwatt levels. In 2000, Peccianti *et al.* reported on nematicon formation in planar cells containing a NLC aligned homogeneously in the presence of an externally applied voltage [5]. They extended the propagation length of nematicons for millimeter levels and found the adequate model for describing nematicon propagation.

The investigation on nematicon was sparse until Conti *et al.* found that the NLC with a pretilt angle induced by an external low-frequency electric field is a kind of strongly nonlocal nonlinear medium and nematicons are a kind of accessible solitons [6–8]. They derived a simplified model and linked nematicons with quadratic solitons. The basic properties on nematicon have been revealed gradually ever since. Among others, we have to mention the interactions between two nematicons [9–12]. Recently, Piccardi *et al.* reported the dark nematicon formation in planar cells filled with dye-doped NLC aligned homeotropically, which can provide an effective negative nonlinearity [13]. The negative nonlinearity is realized through the guest-host interaction.

In this letter, we observed the nematicon formation in planar cells containing a NLC with negative dielectric anisotropy and positive optical anisotropy aligned homogeneously in the presence of an externally applied

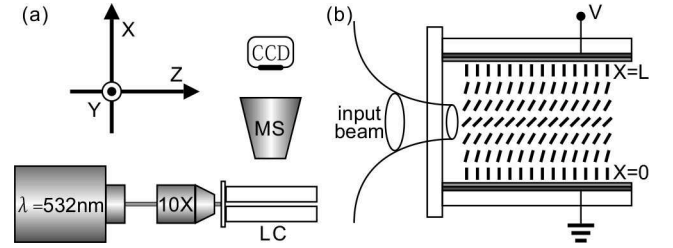


Fig. 1. (Color online) Sketch of the experimental setup (a) and homeotropically aligned nematic liquid crystal cell (b) for the observation of nematicons.

voltage. Following the method in [6], we got a simplified model with a negative Kerr coefficient and an oscillatory periodic response function, which can support bright nematicons. We outlined the connection between the simplified model and the equations describing quadratic solitons [14–17].

We performed a series of experiments to observe the nematicon formation in NLC with negative dielectric anisotropy. The experimental setup is illustrated in Fig. 1(a). A light beam from a Verdi laser was focused by a 10X microscope objective and launched into a 80- μm -thick NLC cell. The configuration of the cell was shown in Fig. 1(b) and the cell was filled with the KY19-008 NLC, whose $n_{\parallel} = 1.726$, $n_{\perp} = 1.496$, average elastic constant $K = 1 \times 10^{-11} \text{ N}$, optical anisotropy $\epsilon_a^{op} = 0.74106$, and dielectric anisotropy $\epsilon_a^{rf} = -5.3$. Owing to the negative dielectric anisotropy, the NLC molecules will try to adjust in a low-frequency applied electrical field in such a manner that the molecule axes turn perpendicular to the direction of the electric field [18]. A microscope and a CCD camera were used to collect the light scattered above the cell during propagation.

One group of experimental results are shown in Figs. 2(a) and 2(c). The launched power and width for each beam is fixed to 4.42 mW and 4 μm when the bias is

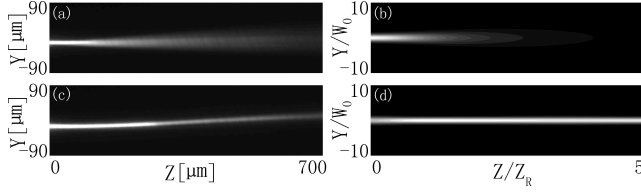


Fig. 2. (Color online) X-polarized e -beam propagation with power $P = 4.42 \text{ mW}$ and width $w = 4 \mu\text{m}$ for different voltage. (a) and (b) Diffraction in the absence of voltage bias. (c) and (d) Nematicon formation in the presence of 3.4 V and 3.01 V bias at 1 kHz, respectively.

changed. Fig. 2(a) shows the linear diffraction in absence of the bias and Fig. 2(c) shows the nematicon formation at $V = 3.4 \text{ V}$ above the Fréedericks threshold [19, 20]

$$V_{fr} = \pi \left(\frac{K}{\epsilon_0 |\epsilon_a^{rf}|} \right)^{1/2}, \quad (1)$$

where ϵ_0 is the vacuum permittivity. Introducing the value of ϵ_a^{rf} and K for KY19-008 NLC, we can get $V_{fr} \approx 1.45 \text{ V}$. The physical mechanism of the nonlinearity in NLC is optically induced molecular reorientation. If the bias is less than the Fréedericks threshold, the NLC molecules will not turn. Then the optical beam will not reorientation the NLC molecules because of the optical electric field is weak under the power in our experiment. Therefore, the optical beam will diffract in absence of the bias and form soliton only if the bias is greater than the Fréedericks threshold. Figs. 2(b) and 2(d) show the numerical results for contrast, which are calculated based on Eqs. (2)–(3) with Gaussian beam as an incident profile.

In the presence of an externally applied (low-frequency) electric field E_{rf} , the evolution of the slowly varying envelope A of a paraxial optical beam linearly polarized along X (an extraordinary light) and propagating along Z can be described by the system

$$2ik \frac{\partial A}{\partial Z} + \nabla_{XY}^2 A + k_0^2 \epsilon_a^{op} (\sin^2 \theta - \sin^2 \theta_0) A = 0, \quad (2)$$

$$2K \left(\frac{\partial^2 \theta}{\partial Z^2} + \nabla_{XY}^2 \theta \right) + \epsilon_0 (\epsilon_a^{rf} E_{rf}^2 + \epsilon_a^{op} \frac{|A|^2}{2}) \sin(2\theta) = 0, \quad (3)$$

where θ is the tilt angle of the NLC molecules, θ_0 is the nadir tilt in the absence of light, $k = k_0 n_e(\theta_0)$ with k_0 the vacuum wavenumber and $n_e(\theta_0) = n_{\perp} n_{\parallel} / (n_{\parallel}^2 \cos^2 \theta_0 + n_{\perp}^2 \sin^2 \theta_0)^{1/2} \approx (n_{\perp}^2 + \epsilon_a^{op} \sin^2 \theta_0)^{1/2}$ the refractive index of the extraordinary light at θ_0 , $\nabla_{XY}^2 = \partial_X^2 + \partial_Y^2$, $\epsilon_a^{rf} = \epsilon_{\parallel} - \epsilon_{\perp} (< 0)$, $\epsilon_a^{op} = n_{\parallel}^2 - n_{\perp}^2 (> 0)$. The term $\partial_Z^2 \theta$ in Eq.(3) was proven to be negligible compared to $\nabla_{XY}^2 \theta$, therefore it can be removed. The homeotropical boundaries and anchoring at the interfaces define $\theta|_{X=0} = \theta|_{X=L} = \pi/2$, where L is the cell thickness. In the absence of light, the pretilt angle $\hat{\theta}$

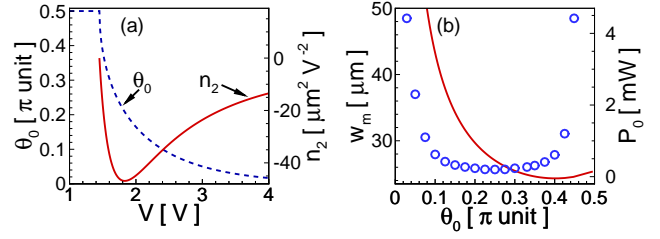


Fig. 3. (Color online) (a) The pretilt angle θ_0 and the Kerr coefficient n_2 of the NLC vs the bias voltage V . (b) The characteristic length w_m (a solid curve) and the critical power of a single soliton (circles) vs the pretilt angle θ_0 . The parameters are for a $80 - \mu\text{m}$ -thick cell filled with the NLC (KY19-008) and the critical power is a numerical result.

is symmetric along X about $X = L/2$ (the cell center) and depends only on X :

$$2K \frac{\partial^2 \hat{\theta}}{\partial X^2} + \epsilon_0 \epsilon_a^{rf} E_{rf}^2 \sin(2\hat{\theta}) = 0. \quad (4)$$

Furthermore, we can set $\theta = \hat{\theta} + (\hat{\theta}/\theta_0)\Phi$, with Φ being the optically induced perturbation. Noting that $\hat{\theta} \approx \theta_0$ and $\partial_X \hat{\theta} \approx 0$ in the middle of the cell when the beam width is far smaller than the cell thickness, we can simplify Eq.(2) and Eq.(3) into the following system, which describes the coupling between A and Φ :

$$2ik \frac{\partial A}{\partial Z} + \nabla_{XY}^2 A + k_0^2 \epsilon_a^{op} \sin(2\theta_0) \Phi A = 0, \quad (5)$$

$$w_m^2 \nabla_{XY}^2 \Phi + \Phi - \frac{2n_0 n_2}{\epsilon_a^{op} \sin(2\theta_0)} |A|^2 = 0, \quad (6)$$

where the parameter w_m ($w_m > 0$ for $|\theta_0| \leq \pi/2$), that is, the characteristic length of the nonlinear response function, reads:

$$w_m = \frac{1}{E_{rf}} \left\{ \frac{2\theta_0 K}{\epsilon_0 |\epsilon_a^{rf}| \sin(2\theta_0) [1 - 2\theta_0 \cot(2\theta_0)]} \right\}^{1/2}, \quad (7)$$

and

$$n_2 = - \frac{(\epsilon_a^{op})^2 \theta_0 \sin(2\theta_0)}{4n_0 |\epsilon_a^{rf}| E_{rf}^2 [1 - 2\theta_0 \cot(2\theta_0)]}. \quad (8)$$

The nonlinear refractive index coefficient n_2 is defined as suggested by Peccianti et al. [19]. It must be noted, however, that n_2 is negative for the liquid crystal with negative dielectric anisotropy, while it is positive for that with positive dielectric anisotropy.

A monotonous function of θ_0 on E_{rf} (or V by $V = E_{rf} L$) is described by Eq. (4). As shown in Fig. 3(a), θ_0 decreases monotonously from $\pi/2$ to 0 with increasing

the bias above the Fréedericks threshold. For E_{rf} higher than the Fréedericksz threshold, the approximation

$$\theta_0 \approx \frac{\pi}{2} \left(\frac{E_{fr}}{E_{rf}} \right)^3 \quad (9)$$

is satisfactory, where $E_{fr} = V_{fr}/L$. Therefore, we can clearly see from Eqs.(7) and (8) that w_m and n_2 are determined by V or θ_0 for a given NLC cell configuration. As shown in Fig. 2, w_m and n_2 changes nonmonotonously with increasing θ_0 and V , respectively. Both of them have a minimum value. There we also show the relation of the critical power P_0 on θ_0 calculating numerically based on Eqs. (2)–(3) with Gaussian beam as an incident profile.

Introducing the normalization that $x = X/w_m$, $y = Y/w_m$, $z = Z/(kw_m^2)$, $u = A/A_0$, $\phi = \Phi/\Phi_0$, where $A_0 = [8K/k_0^2 w_m^4 \epsilon_0 \epsilon_a^{op2} \sin^2(2\theta_0)]^{1/2}$, $\Phi_0 = 2/[k_0^2 w_m^2 \epsilon_a^{op} \sin(2\theta_0)]$, we have the dimensionless system,

$$i \frac{\partial u}{\partial z} + \frac{1}{2} \nabla_{\perp}^2 u + \phi u = 0, \quad (10)$$

$$\nabla_{\perp}^2 \phi + \phi = -|u|^2, \quad (11)$$

where $\nabla_{\perp}^2 = \partial_x^2 + \partial_y^2$. We consider a planar geometry with the boundary condition $\phi|_{x=0,l} = 0$. Eq.(11) has a particular solution in the form of a convolution integral of $|u|^2$ with the function R :

$$\phi(x, y) = \int_0^l \int_{-\infty}^{+\infty} R(x, y; x', y') |u(x', y')|^2 dx' dy'. \quad (12)$$

Here

$$R(x, y) = \sum_{m=1}^{\infty} a_m(y) \sin \frac{m\pi x}{l}, \quad (13)$$

with

$$a_m = \begin{cases} -\frac{\sqrt{\varsigma_m}}{l} \sin \frac{m\pi x'}{l} \sin \left| \frac{y-y'}{\sqrt{\varsigma_m}} \right| & m < l/\pi, \\ \frac{\sqrt{|\varsigma_m|}}{l} \sin \frac{m\pi x'}{l} \exp[-\left| \frac{y-y'}{\sqrt{|\varsigma_m|}} \right|] & m > l/\pi, \end{cases} \quad (14)$$

where $\varsigma_m = 1/[1 - (\frac{m\pi}{l})^2]$. This means that the tilt angle will be sine-oscillatory for a gaussian input beam when the planar thickness $l > \pi$ and exponential-decay $l < \pi$. From Eqs.(1), (7), and (9), we can get

$$l = \pi^{4/3} (2\theta_0)^{-5/6} \sin^{1/2}(2\theta_0) [1 - 2\theta_0 \cot(2\theta_0)]^{1/2}. \quad (15)$$

We can see that l is changed only with θ_0 . The maximum of l is comparable with π . Therefore, the tilt angle of original model Eqs. (2)–(3) is not sine-oscillatory when $l > \pi$, although the tilt angle of simplified model Eqs. (10)–(11) is.

We found many solitons from Eqs. (10)–(11) using numerical iteration method. It was found that the soliton profile is changed with the sample size l_x and l_y and the propagation constant β . Figs. 4 show the two group of typical soliton profile w and the corresponding tilt angle ϕ . When $l_x < \pi$ (or $l_y < \pi$), ϕ is exponential-decay, as

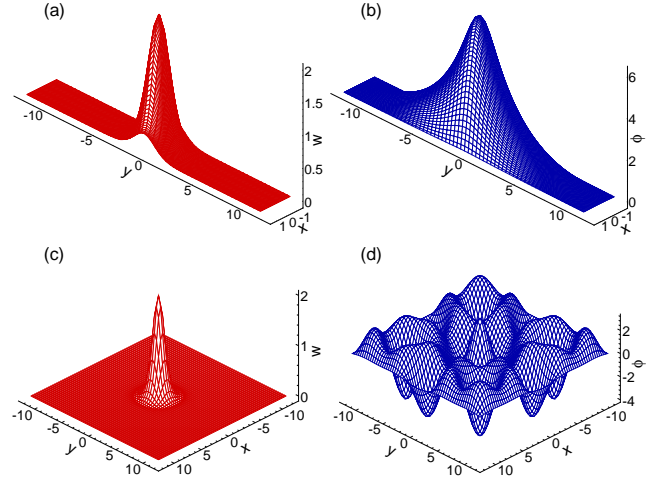


Fig. 4. (Color online) The soliton profile [(a) and (c)] and tilt angle [(b) and (d)] for different sample size l_x and l_y and propagation constant β . (a)–(b) $l_x = 3$, $l_y = 23.6$, $\beta = 4.3099$. (c)–(d) $l_x = l_y = 23.6$, $\beta = 1.4326$.

shown in Fig. 4(b). When $l_x > \pi$ (or $l_y > \pi$), ϕ is sine-oscillatory, as shown in Fig. 4(d). This result agrees with the above analysis of Greens function method.

In summary, we observed experimentally the nematicon formation in the planar cell containing the nematic liquid crystal with negative dielectric anisotropy, aligned homeotropically in the presence of an externally applied voltage. We gave a theoretical model describing nematicon propagation, based on which we investigated the evolution of a Gaussian beam. And then we derived a simplified model, from which we found many bright nematicon with a sine-oscillatory response function and a negative Kerr coefficient.

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